CET (PG)-2018

Sr. No. : ...110245...............

Booklet Series Code : A

Important : Please consult your Admit Card / Roll No. Slip before filling your Roll Number on the Test Booklet and Answer Sheet.

	(In Figures)	*(In Words)*
Roll No.		....

O.M.R. Answer Sheet Serial No.

Signature of the Candidate : ....

Subject : MATHEMATICS

Time : 90 Minutes] **[Maximum Marks : 75**

Number of Questions : 75] **[Total No. of Printed Pages : 20**

DO NOT OPEN THE SEAL ON THE BOOKLET UNTIL ASKED TO DO SO

INSTRUCTIONS :

1. Write your Roll No. on the Question Booklet and also on the OMR Answer Sheet in the space provided and nowhere else.
2. Enter the Subject and Series Code of Question Booklet on the OMR Answer Sheet. Darken the corresponding bubbles with **Black Ball Point / Black Gel Pen.**
3. Do not make any identification mark on the Answer Sheet or Question Booklet.
4. To open the Question Booklet remove the paper seal gently when asked to do so.
5. Please check that this Question Booklet contains **75** questions. In case of any discrepancy, inform the Assistant Superintendent within 10 minutes of the start of test.
6. Each question has four alternative answers (A, B, C, D) of which only one is correct. For each question, darken only one bubble (A or B or C or D), whichever you think is the correct answer, on the Answer Sheet with **Black Ball Point / Black Gel Pen.**
7. If you do not want to answer a question, leave all the bubbles corresponding to that question blank in the Answer Sheet. No marks will be deducted in such cases.
8. Darken the bubbles in the OMR Answer Sheet according to the Serial No. of the questions given in the Question Booklet.
9. Negative marking will be adopted for evaluation i.e., 1/4th of the marks of the question will be deducted for each wrong answer. A wrong answer means incorrect answer or wrong filling of bubble.
10. For calculations, use of simple log tables is permitted. Borrowing of log tables and any other material is not allowed.
11. For rough work only the sheets marked "Rough Work" at the end of the Question Booklet be used.
12. The Answer Sheet is designed for **computer evaluation**. Therefore, if you do not follow the instructions given on the Answer Sheet, it may make evaluation by the computer difficult. **Any resultant loss to the candidate on the above account, i.e., not following the instructions completely, shall be of the candidate only.**
13. After the test, hand over the Question Booklet and the Answer Sheet to the Assistant Superintendent on duty.
14. In no case the Answer Sheet, the Question Booklet, or its part or any material copied/noted from this Booklet is to be taken out of the examination hall. Any candidate found doing so, would be expelled from the examination.
15. A candidate who creates disturbance of any kind or changes his/her seat or is found in possession of any paper possibly of any assistance or found giving or receiving assistance or found using any other unfair means during the examination will be expelled from the examination by the Centre Superintendent/Observer whose decision shall be final.
16. **Telecommunication equipment such as pager, cellular phone, wireless, scanner, etc., is not permitted inside the examination hall. Use of calculator is not allowed.**

SEAL

1. If the product of two roots of $x^4 + px^3 + qx^2 + rx + s = 0$ is equal to the product of other two, then :

(A) $r^2 = p^2s$

(B) $r = p^2s$

(C) $r^2 = qs$

(D) $r = q^2s$

2. If α, β, γ are the roots of $x^3 - x - 1 = 0$, then the value of $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ is :

(A) -7

(B) 7

(C) 5

(D) -4

3. If α, β, γ are cube roots of p, $p < 0$ then for any x, y, z the value of $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ are :

(A) w, w^2

(B) $-w, -w^2$

(C) $1, -1$

(D) $1 - w, 1 - w^2$

4. If the set A and B are defined as $A = \{(x, y) : y = 1/x, 0 \neq x \in R\}$, $B = \{(x, y) : y = -x, x \in R\}$, then :

(A) $A \cap B = A$

(B) $A \cap B = B$

(C) $A \cap B = \phi$

(D) $A \cup B = B$

5. The value of $\lim_{n \to \infty} \frac{1}{n}\left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{n-2} + \ldots + \left(\frac{1}{n}\right)\left(\frac{1}{2}\right)\right]$ is :

(A) 0

(B) Does not exist

(C) 1

(D) 2

6. **The angle between the lines joining the origin to the points of intersection of the straight line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$:**

(A) $\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (B) $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(C) $\tan^{-1}\left(\frac{3}{2\sqrt{2}}\right)$ (D) $\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$

7. **Let ρ be the relation on the set R of all real numbers defined by setting $a \rho b$ iff $|a-b| \le \frac{1}{2}$.**

Then, ρ is :

(A) Reflexive and symmetric but not transitive

(B) Symmetric and transitive but not reflexive

(C) Transitive but neither reflexive nor symmetric

(D) Reflexive, symmetric and transitive

8. **The series $\frac{1^2}{2^2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}x + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2}x^2 + ...$ is :**

(A) Convergent, if $x < 1$ and divergent, if, $x \ge 1$

(B) Convergent, if $x > 1$ and divergent, if, $x \le 1$

(C) Convergent, if $x \le 1$ and divergent, if, $x > 1$

(D) Convergent, if $x \ge 1$ and divergent, if, $x < 1$

9. **The series $\sum \frac{1}{n^p + n^{-p}}$ converges, if :**

(A) $|p| > 1$ (B) $p = 1$

(C) $|p| < 1$ (D) None

10. **The series** $\frac{3}{5}x^4 + \frac{8}{10}x^6 + \frac{15}{17}x^8 + \ldots\ldots + \frac{n^2-1}{n^2+1}x^{2n} + \ldots\ldots$ **is :**

(A) Convergent, if $x^2 \geq 1$ and divergent, if, $x^2 < 1$

(B) Convergent, if $x^2 \leq 1$ and divergent, if, $x^2 > 1$

(C) Convergent, if $x^2 < 1$ and divergent, if, $x^2 \geq 1$

(D) Convergent, if $x^2 > 1$ and divergent, if, $x^2 \leq 1$

11. **The function** $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ **has :**

(A) Removable discontinuity at 0

(B) Non removable discontinuity at 0

(C) Mixed discontinuity at 0

(D) Continuity at 0

12. **If** $x_1 = a$ **and** $x_{n+1} = \sqrt{b + c\,x_n}$**, with** $a > 1, b > 1, c > 1$**, for all** $n \in \mathbb{N}$**, then the sequence** $\{x_n\}$ **converges to the positive root of the equation :**

(A) $x^2 - cx - b = 0$

(B) $x^2 + cx - b = 0$

(C) $x^2 + cx + b = 0$

(D) $x^2 - cx + b = 0$

13. **The entire length of cardioid** $r = a(1 + \cos\theta)$ **is :**

(A) 8a

(B) 4a

(C) 6a

(D) 2a

14. **The volume of the solid bounded by the surfaces** $z = 0, 3z = x^2 + y^2$ **and** $x^2 + y^2 = 9$ **:**

(A) 27/2

(B) $27\pi/4$

(C) 9π

(D) $27\pi/2$

15. By changing the order of integration $\int_0^a \int_0^x \frac{\cos y \, dy}{\sqrt{(a-x)(a-y)}} dx$ is equal to :

(A) $\int_0^a dy \int_y^a \frac{\cos y \, dx}{\sqrt{(a-x)(a-y)}}$

(B) $\int_0^a dx \int_y^a \frac{\cos y \, dy}{\sqrt{(a-x)(a-y)}}$

(C) $\int_0^a dx \int_a^y \frac{\cos y \, dy}{\sqrt{(a-x)(a-y)}}$

(D) $\int_0^a dx \int_y^a \frac{\cos (x+y) \, dy}{\sqrt{(a^2-x-y)}}$

16. A function f defined such that for all real x, y, f(x + y) = f(x) f(y), and f(x) = 1 + xg(x) where $\lim_{x \to 0} g(x) = 1$. What is $\frac{d}{dx} f(x)$ equals to ?

(A) g(x)

(B) f(x)

(C) g'(x)

(D) g(x) + xg'(x)

17. The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is :

(A) 1

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) 2

18. Let a be a complex number such that | a | < 1 and $z_1, z_2, \ldots$ be vertices of a polygon such that $z_k = 1 + a + a^2 + \ldots + a^{k-1}$. Then the vertices of the polygon lie within a circle :

(A) $|z - a| = a$

(B) $\left| z - \frac{1}{1-a} \right| = a$

(C) $\left| z - \frac{1}{1-a} \right| = \frac{1}{|1-a|}$

(D) $|z - (1-a)| = |1-a|$

19. The value of set of 'c' of Lagrange's mean value theorem, if $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \frac{1}{2}$ is :

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{6-\sqrt{21}}{6}$

(D) $\frac{6+\sqrt{21}}{6}$

20. A, B, C, D are four points such that $\overrightarrow{AB} = m(2\hat{i} - 6\hat{j} + 2\hat{k}), \overrightarrow{BC} = \hat{i} - 2\hat{j}$ and $\overrightarrow{CD} = n(-6\hat{i} + 15\hat{j} - 3\hat{k})$. CD intersects AB at some point E such that $m > \frac{1}{2}$ and $n > \frac{1}{3}$.

The area of the triangle BCE is :

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{6}}{3}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\frac{\sqrt{6}}{2}$

21. The equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $P(1, -2, 1)$ and also cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$:

(A) $x^2 + y^2 + z^2 + 10x + 7y - 5z + 13 = 0$

(B) $x^2 + y^2 + z^2 + 10x + 7y - 13z + 5 = 0$

(C) $x^2 + y^2 + z^2 + 7x + 10y + 13z - 5 = 0$

(D) $x^2 + y^2 + z^2 + 7x + 10y - 5z + 13 = 0$

22. A particle moves along a straight line and its velocity at any time t is given by the equation $v = \pi \cos\left(\frac{\pi}{2}t\right)$. Given the motion is simple harmonic motion. The total distance covered by the particle during the interval from $t = 0$ to $t = 3$ seconds is :

(A) 2 units

(B) 4 units

(C) 8 units

(D) 6 units

23. The series $\sum_{k=0}^{\infty} \frac{k^2+3k+1}{(k+2)!}$ converges to :

(A) 1

(B) 1/2

(C) 2

(D) 3

24. The set of all positive rational numbers forms an abelian group under the composition define by :

(A) $a * b = \frac{ab}{2}$

(B) $a * b = \frac{a+b}{2}$

(C) $a * b = \frac{2a}{b}$

(D) $a * b = \frac{a-b}{2}$

25. The number of elements of order 5 in the group $Z_{25} \oplus Z_5$ is :

(A) 5

(B) 10

(C) 24

(D) 25

26. In a group $GL(2, Z_7)$, the inverse of $A = \begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 6 \\ 3 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

27. If $G = \{1, -1, i, -i\}$ is a multiplicative group, then order of $-i$ is :

(A) One

(B) Two

(C) Three

(D) Four

28. If H_1 and H_2 are two subgroups of G, then following is also a subgroup of G :

(A) $H_1 \cap H_2$

(B) $H_1 \cup H_2$

(C) $H_1 H_2$

(D) None of these

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29. **What is the dimension of the vector space formed by the solution of the system of the following equations ?**

$$x_1 + x_2 + x_3 = 0,\ x_1 + 2x_2 = 0,\ x_2 - x_3 = 0$$

(A) 1 (B) 2

(C) 3 (D) 0

30. **The linear transformation** $T(x, y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ **can be written as :**

(A) $T(x, y) = (x, y)$ (B) $T(x, y) = (y, x)$

(C) $T(x, y) = (x, -y)$ (D) $T(x, y) = (-y, -x)$

31. **The system of equation $2x + y = 5$, $x - 3y = -1$, $3x + 4y = k$ is consistent, when k is :**

(A) 0 (B) 13

(C) 1 (D) 10

32. **The eigen values of a 3×3 real matrix A are 1, 2 and 3. Then :**

(A) $A^{-1} = -\frac{1}{6}A^2$ (B) $A^{-1} = \frac{1}{6}(7I - A^2)$

(C) $A^{-1} = -\frac{1}{6}(7I - A^2)$ (D) $A^{-1} = \frac{1}{6}(-7I - A^2)$

33. **If** $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$**, then A^{50} is :**

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 100 & 1 & 0 \\ 100 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 50 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$

34. **The minimal polynomial associated with the matrix $\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ is :**

(A) $x^3 - x^2 - 2x - 3$

(B) $x^3 - x^2 + 2x - 3$

(C) $x^3 - x^2 - 3x - 3$

(D) $x^3 - x^2 + 3x - 3$

35. **If mean and variance of a random variable X having a binomial distribution are 4 ar respectively. Then, P(X > 6) is equal to :**

(A) $\frac{1}{256}$

(B) $\frac{3}{256}$

(C) $\frac{9}{256}$

(D) $\frac{7}{256}$

36. **If X is exponentially distributed with parameter λ, find the value of k such that $\frac{P(X > k)}{P(X \le k)} = a$:**

(A) $\frac{1}{\lambda}\log\left(\frac{1+a}{a}\right)$

(B) $\frac{1}{\lambda}\log\left(\frac{1-a}{a}\right)$

(C) $\frac{1+a}{a}$

(D) a

37. **Let A and B be two events such that $P(\overline{A \cup B}) = 1/6, P(A \cap B) = 1/4$ and $P(\overline{A}) = 1/4$, wher stands for complement of event A. Then, events A and B are :**

(A) Equally likely and mutually exclusive

(B) Equally likely but not independent

(C) Independent but not equally likely

(D) Mutually exclusive and independent

38. **The Newton-Raphson algorithm for the function $\frac{1}{a}$ will be :**

(A) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$

(B) $x_{k+1} = x_k + \frac{a}{2}x_k^2$

(C) $x_{k+1} = 2x_k - ax_k^2$

(D) $x_{k+1} = x_k - \frac{a}{2}x_k^2$

39. The Gauss-Seidal method gives results faster when the pivotal element are :

(A) Smaller than other coefficients

(B) Larger than other coefficients

(C) Equal to other coefficients

(D) One

40. The Newton divided difference polynomial which interpolate the data f(0) = 1, f(1) = 3, f(3) = 55 is :

(A) $8x^2 + 6x + 1$

(B) $8x^2 - 6x + 1$

(C) $8x^2 + 6x - 1$

(D) $8x^2 + 6x - 3$

41. Let α be the actual root of f(x) = 0 and it requires n iterations to find the approximated root β, such that $|\alpha - \beta| < 10^{-3}$ using bisection method of finding the roots. Use the starting interval [1, 11] the value n will be :

(A) $n \geq \frac{2}{\log_{10}(2)} + 1$

(B) $n \geq \frac{4}{\log_{10}(2)} + 1$

(C) $n \geq \frac{4}{\log_{10}(2)}$

(D) $n \geq \frac{2}{\log_{10}(2)}$

42. The second order Runge-Kutta method is applied to the initial value problem $y' = -y$, $y(0) = y_0$, with step size h. Then, y(h) is :

(A) $y_0(h - 1)^2$

(B) $\frac{y_0}{2}(h^2 - 2h + 2)$

(C) $\frac{y_0}{6}(h^2 - 2h + 2)$

(D) $y_0\left(1 - h + \frac{h}{2} + \frac{h^3}{6}\right)$

43. If $f_n(x)$ be a sequence of real valued function and converges uniformly to f(x). Then which one of the following is correct ?

(A) If each $f_n(x)$ is continuous $\Rightarrow$ f is continuous

(B) If each $f_n(x)$ is continuous $\Rightarrow$ f may not be continuous

(C) If each $f_n(x)$ is differentiable $\Rightarrow$ f is differentiable

(D) f is not continuous

44. **The sequence of real-valued functions $f_n(x) = x^n$, $\forall x \in [0, 1] \cup \{2\}$, is :**

(A) pointwise convergent but not uniformly convergent

(B) uniformly convergent

(C) not bounded

(D) bounded but not pointwise convergent

45. **If we expand sin x by Taylor's series about $\frac{\pi}{2}$, then a_2, a_3, a_4 are :**

(A) $-\frac{1}{2}, 0, \frac{1}{24}$

(B) $-\frac{1}{2}, 0, -\frac{1}{24}$

(C) $\frac{1}{2}, 0, \frac{1}{24}$

(D) $0, -\frac{1}{7!}, 0$

46. **The series $\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{2n-1}\right)$ is :**

(A) Convergent

(B) Divergent

(C) Unbounded

(D) None of these

47. **Which one is uniformly continuous in $[0, \infty)$?**

(A) x^2

(B) sin x

(C) $\sin x^2$

(D) None of these

48. **For $(x, y) \in R^2$, let $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$, then :**

(A) f is continuous at (0, 0) and the partial derivatives f_x, f_y exist at every point of R^2

(B) f is discontinuous at (0, 0) and f_x, f_y does not exists at every point of R^2

(C) f is discontinuous at (0, 0) and f_x, f_y exists at (0, 0)

(D) None of the above

49. **At t = 0, the function $f(t) = \frac{\sin t}{t}$ has :**

(A) A minimum

(B) A discontinuity

(C) A point of inflection

(D) A maximum

50. **Let $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$, then :**

(A) f_x does not exist at origin

(B) f_y exists at origin

(C) f_y does not exist at origin

(D) None

51. **Given the function $f(x, y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$:**

(A) has maximum value at origin

(B) has minimum value at origin

(C) has neither maximum nor minimum value at origin

(D) has maximum value but no minimum value at origin

52. **Let $I = \int_0^{\pi/2} \log \sin x \, dx$, then :**

(A) I diverges at $x = 0$

(B) I converges and equals to $-\pi \log 2$

(C) I diverges at $x = \frac{\pi}{4}$

(D) I converges and equals to $-\frac{\pi}{2} \log 2$

53. **Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = \begin{cases} \frac{xy}{x^2+y} & ; x^2 \neq -y \\ 0 & ; x^2 = -y \end{cases}$ then :**

(A) directional derivative does not exist at (0, 0)

(B) each directional derivative exists at (0, 0) but f is not continuous

(C) f is differentiable at (0, 0)

(D) f is continuous at (0, 0)

54. **If $u = x^2$ and $v = y^2$, then $\frac{\partial(u, v)}{\partial(x, y)}$ is equal to :**

(A) 5xy
(B) 4xy
(C) 2xy
(D) xy

55. **Which of the following is not equal to J(x, y) (Jacobian) ?**

(A) $-J(y, x)$
(B) $J(y, -x)$
(C) $J(y, x)$
(D) $J(-y, x)$

56. **If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is :**

(A) $\frac{xy}{x^2 + y^2}$
(B) $\frac{xy^2}{x^2 + y^4}$
(C) $\frac{x^2y}{x^2 + y^4}$
(D) $\frac{x^2y^2}{x^2 + y^4}$

57. **The particular integral of $(D^2 - 2D + 4)y = e^x \cos x$ is :**

(A) $\cos x$
(B) $\sin x$
(C) $\frac{1}{2}e^x \cos x$
(D) $\frac{1}{2}e^x \sin x$

58. **The solution of the simultaneous equations $\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$ and $\frac{dy}{dt} + 5x + 3y = 0$ is :**

(A) $c_1 \cos x + c_2 \sin x - (\log \cos x) \cos x + x \sin x$
(B) $c_1 \cos x + c_2 \sin x + x \cos x + (\log \cos x) \sin x$
(C) $c_1 \cos x + c_2 \sin x + (\log \cos x) \cos x - x \sin x$
(D) $c_1 \cos x + c_2 \sin x + (\log \cos x) \cos x + x \sin x$

59. **Which of the following transformation reduce the differential equation** $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$ **into the form** $\frac{du}{dx}+P(x)u=Q(x)$ **?**

(A) $u=\log z$

(B) $u=\frac{1}{\log z}$

(C) $u=e^z$

(D) $u=(\log z)^2$

60. **The integrating factor for the differential equation** $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$**, is :**

(A) $\frac{1}{x+1}$

(B) $x+1$

(C) $\frac{1}{x^2+1}$

(D) x^2+1

61. **The integrating factor of** $(x^7y^2+3y)dx+(3x^8y-x)dy=0$ **is** x^my^n **:**

(A) $m=-7, n=2$

(B) $m=-1, n=7$

(C) $m=-7, n=1$

(D) $m=-7, n=-2$

62. **The orthogonal trajectories of the family of coaxial circles** $x^2+y^2+2g\,x+c=0$**, where g is a parameter, is :**

(A) $2xy\frac{dx}{dy}-x^2=c+y^2$

(B) $2xy\frac{dy}{dx}-x^2=-c-y^2$

(C) $2xy\frac{dy}{dx}+x^2=-c-y^2$

(D) $2xy\frac{dy}{dx}+x^2=c+y^2$

63. **Find the singular solution of the differential equation** $y=(y')^2-3xy'+3x^2$**. The general solution of the equation is known and given by the function** $y=Cx+C^2+x^2$ **:**

(A) $y=\frac{4}{3}x^3$

(B) $y=\frac{3}{4}x^3$

(C) $y=\frac{4}{3}x$

(D) $y=\frac{3}{4}x^2$

64. Let $y_1(x)$ and $y_2(x)$ be linearly independent solution of the differential equation $y'' + P(x)y' + Q(x)y = 0$, where $P(x)$ and $Q(x)$ are continuous functions on an interval I. Then $y_3(x) = ay_1(x) + by_2(x)$ and $y_4(x) = cy_1(x) + dy_2(x)$ are linearly independent solutions of the given differential equation if :

(A) $ad = bc$

(B) $ac = bd$

(C) $ad \neq bc$

(D) $ac \neq bd$

65. Let $y_1(x)$ and $y_2(x)$ be two solution of $(1 - x^2)\, y'' - 2xy' + (\sec x)y = 0$ with Wronskian $W(x)$. If $y_1(0) = 1$, $\left(\frac{dy_1}{dx}\right)_{x=0} = 0$ and $W\left(\frac{1}{2}\right) = \frac{1}{3}$, then $\left(\frac{dy_2}{dx}\right)_{x=0}$ equals :

(A) 1/4

(B) 1

(C) 3/4

(D) 4/3

66. Transform the equation $14x^2 - 4xy + 11y^2 - 36x + 48y + 41 = 0$ to rectangular axis through the point $(1, -2)$ inclined at an angle $\tan^{-1}\left(\frac{1}{2}\right)$ to the original axis :

(A) $x^2 + 2y^2 = 5$

(B) $3x^2 + 2y^2 - 5 = 0$

(C) $3x^2 + y^2 = 5$

(D) $3x^2 + 2y^2 + 5 = 0$

67. The Laplace transform of $e^{-2t} \cos(4t)$ is :

(A) $\frac{s+2}{(s+2)^2 + 16}$

(B) $\frac{s+2}{(s+2)^2 - 16}$

(C) $\frac{s-2}{(s-2)^2 - 16}$

(D) $\frac{s-2}{(s-2)^2 + 16}$

68. If $F(s)$ is the Laplace transform of function $f(t)$, then Laplace transform of $\int_0^t f(\tau)\, d\tau$ is :

(A) $\frac{1}{s}F(s)$

(B) $\frac{1}{s}F(s) - f(0)$

(C) $sF(s) - f(0)$

(D) $\int F(s)\, ds$

69. Consider $M_1 = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$, $M_2 = \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}$, $M_3 = \begin{bmatrix} 5 & -6 \\ -3 & -2 \end{bmatrix}$, and $M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ of $M_{2\times2}(\mathbb{R})$, then :

(A) $\{M_2, M_3, M_4\}$ is linearly independent

(B) $\{M_1, M_2, M_3\}$ is linearly dependent

(C) $\{M_1, M_3, M_4\}$ is linearly independent

(D) $\{M_1, M_2, M_4\}$ is linearly independent

70. The moments of a system of coplanar forces about the points (1, 0), (0, 2) and (2, 3) referred to rectangular axes are G_1, G_2, G_3, respectively. If θ be the inclination of the resultant to the axis of x, then $\tan\theta$ is given by :

(A) $\dfrac{G_1 - 3G_2 + 2G_3}{2G_1 - G_2 - G_3}$

(B) $\dfrac{G_1 - 3G_2 + 2G_3}{G_1 - 2G_2 - G_3}$

(C) $\dfrac{G_1 - 2G_2 + 2G_3}{G_1 - 2G_2 - G_3}$

(D) $\dfrac{G_1 - 3G_2 + 3G_3}{G_1 - 2G_2 - G_3}$

71. The resultant of the forces P and Q is R. If Q be doubled, R is doubled. If Q be reversed, R is again doubled. Then P : Q : R is given as :

(A) $\sqrt{2}:\sqrt{2}:\sqrt{3}$

(B) $\sqrt{3}:\sqrt{2}:\sqrt{2}$

(C) $\sqrt{2}:\sqrt{3}:\sqrt{2}$

(D) $\sqrt{2}:\sqrt{3}:\sqrt{3}$

72. The train is moving at a speed of 44 km/hour. A stone strikes it at right angles with a speed of 33 km/hour. The direction of the velocity of the stone with which it appears to strike the passanger sitting in the train is :

(A) $\pi - \tan^{-1}\dfrac{3}{5}$

(B) $\pi - \tan^{-1}\dfrac{2}{3}$

(C) $\pi - \tan^{-1}\dfrac{4}{5}$

(D) $\pi - \tan^{-1}\dfrac{3}{4}$

73. If $u = x^2 + y^2 + z^2$ and $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$ then $\text{div}(u\vec{v})$:

(A) 3u

(B) 4u

(C) 5u

(D) 6u

74. The value of $\iint_S (ax^2 + by^2 + cz^2)ds$ over the sphere $S : x^2 + y^2 + z^2 = 1$ is given by :

(A) $\frac{4}{3}\pi^3 \sqrt{a^2 + b^2 + c^2}$

(B) $\frac{1}{2}\pi \sqrt{a^2 + b^2 + c^2}$

(C) $\frac{1}{2}\pi^2 \sqrt{a^2 + b^2 + c^2}$

(D) $\pi^2 \sqrt{\frac{a^2 + b^2 + c^2}{2}}$

75. The Integral $\int_0^\infty \sin x \, dx$:

(A) Exists and equals 0

(B) Exists and equals 1

(C) Exists and equals –1

(D) Does not exist