Question Booklet Series: **A** *Question Booklet Serial No.* 100042

Ph.D. Entrance Test – 2018

Subject: Mathematics

Paper – I

Important: *Please consult your Admit Card/Roll No. slip before filling your Roll Number on the Test Booklet and Answer Sheet.*

Roll No. *In Figure* *In Words*

O.M.R. Answer Sheet Serial No.

Signature of Candidate: ____________________ *Signature of Invigilator:* ________________

Time: 60 Minutes **Number of Questions: 50** **Maximum Marks: 50**

DO NOT OPEN THE SEAL ON THE BOOKLET UNTIL ASKED TO DO SO.

INSTRUCTIONS:

1. Write your Roll No. on the Questions Booklet and also on the OMR Answer Sheet in the space provided and nowhere else.
2. Enter the Question Booklet Serial No. on the OMR Answer Sheet. Darken the corresponding bubbles with **Black Ball Point/Black Gel Pen**.
3. Do not make any identification mark on the Answer Sheet or Question Booklet.
4. Please check that this Question Booklet contains **50** Questions. In case of any discrepancy, inform the Assistant Superintendent within 10 minutes of the start of Test.
5. Each question has four alternative answer (A,B,C,D) of which only one is correct. For each question, darken only one bubble (A or B or C or D), whichever you think is the correct answer, on the Answer Sheet with **Black Ball Point/Black Gel Pen. <u>There shall be no negative marking for wrong answers.</u>**
6. If you do not want to answer a question, leave all the bubbles corresponding to that question blank in the Answer Booklet. No marks will be deducted in such cases.
7. Darken the bubbles in the OMR Answer Sheet according to the Serial No. of the question given in the Question Booklet.
8. If you want to change an already marked answer, erase the shade in the darkened bubble completely.
9. For rough work only the blank sheet at the end of the Question Booklet be used.
10. The Answer Sheet is designed for computer evaluation. Therefore, if you do not follow the instructions given on the Answer Sheet, it may make evaluation by the computer difficult. **Any resultant loss to the candidate on the above account, i.e. not following the instructions completely, shall be of the candidate only.**
11. After the test, hand over the Question Booklet and the Answer Sheet to the Assistant Superintendent on duty.
12. In no case the Answer Sheet, the Question Booklet, or its part or any material copied/noted from this Booklet is to be taken out of the examination hall. Any candidate found doing so would be expelled from the examination.
13. A candidate who creates disturbance of any kind or changes his/her seat or is found in possession of any paper possibly of any assistant or found giving or receiving assistant or found using any other unfair means during the examination will be expelled from the examination by the Centre Superintendent/Observer whose decision shall be final.
14. **Communication equipment such as mobile phones, pager, wireless set, scanner, camera or any electronic/digital gadget etc., is not permitted inside the examination hall. Use of calculators is not allowed.**
15. The candidates will not be allowed to leave the Examination Hall/Room before the expiry of the allotted time.

(1068)

Question 1 The Newton-Raphson method formula for finding square root of a positive real number R from the equation $x^2 = R$ is

(a) $x_{i+1} = \dfrac{x_i}{2}$ (b) $x_{i+1} = \dfrac{3x_i}{2}$

(c) $x_{i+1} = \dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$ (d) $x_{i+1} = \dfrac{1}{2}\left(3x_i - \dfrac{R}{x_i}\right)$

Question 2 The minimal polynomial of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ is

(a) $x(x-4)$ (b) $x^2(x-4)$

(c) $x^3(x-4)$ (d) $x^4(x-4)$

Question 3 Assuming initial interval of [1,5], the value of $te^{-t} - 0.3 = 0$ at the end of second interation using bisection method is

(a) 0 (b) 1.5

(c) 2 (d) 3

Question 4 Let X be the set of all irrational numbers with discrete metric, then which of the following is true

(a) X is complete (b) X is compact

(c) X is connected (d) X is bounded

Question 5 Let R be a ring. If R[x] is a principal ideal domain, then R is neccesarily a

(a) unique factorization domain (b) principal ideal domain

(c) Euclidean domain (d) field

Question 6 Let $\{a_n\}$ be a sequence with $a_n = \left(1 + \dfrac{1}{n}\right)^{n+1}$; $n = 1, 2, 3, ...$ then which of the following is true

(a) $\{a_n\}$ is increasing and convergent (b) $\{a_n\}$ is increasing but not convergent

(c) $\{a_n\}$ is decreasing and convergent (d) $\{a_n\}$ is decreasing but not convergent

Question 7 Which of the following is not true

(a) The set of rational numbers is countable

(b) For every real number r there is a sequence of rational numbers converging to r

(c) Every non-empty subset of rational numbers $\mathbb{Q}$ which is bounded above has a least upper bound in $\mathbb{Q}$

(d) For every subset A of $\mathbb{R}$, for every element of A, there is an integer greater than that element of A

Question 8 Let X be a metric space. Let A $\subseteq$ X. Then which of the following statements is true

(a) If A is compact in X, then A is closed in X
(b) If A is closed in X, then A is compact in X
(c) If A $\subseteq$ X is open, then A has no limit points
(d) If A has an empty interior, then all points of A are isolated

Question 9 If A is a 5×7 matrix of rank 4, then the number of Linearly independent solutions of $AX = 0$ is

(a) 5 (b) 4
(c) 3 (d) 1

Question 10 If A and B are $n \times n$ matrices and $det(A)$ denotes the determinant of A, then which of the following is false:

(a) $det(AB) = det(A)\ det(B)$ (b) $det(A^{-1}BA) = det(B)$ if $A$is invertible
(c) $det(A^2B^2) = det(ABAB)$ (d) if α is a real number, then $det(\alpha A) = \alpha det(A)$

Question 11 If x and y are columns of length m and n respectively, $m > n$, then the rank of xy' is

(a) 1 (b) m
(c) n (d) $m - n$

Question 12 For $\dfrac{z - \sin z}{z^3}$, the point $z = 0$ is

(a) a pole (b) a removable singularity
(c) an essential singularity (d) none of the above

Question 13 If α, β, γ are the cube roots of $1 + \iota$ then $\alpha + \beta + \gamma$ equals

(a) 0 (b) 1
(c) ι (d) $1 + \iota$

Question 14 The inverse point of $1 + \iota$ with respect to the circle $|z - 1| = 2$ is

(a) $1 - \iota$ (b) $1 + 4\iota$
(c) $1 + 2\iota$ (d) $-1 - \iota$

Question 15 The function $f(z) = \overline{z},\ z \in \mathbb{C}$ satisfies which of the following

(a) it is analytic everywhere (b) it is analytic everywhere except at $z = 0$
(c) it is analytic nowhere except at $z = 0$ (d) it is nowhere analytic

Question 16 Let $a, b \in G$, where G is a group. Which of the following is not true in general

(a) $(ab)^{-1} = b^{-1}a^{-1}$ (b) $(a^{-1})^{-1} = a$
(c) each $a \in G$ has a unique inverse (d) $(ab)^2 = a^2b^2$

Question 17 The zero divisors in $\mathbb{Z}_8$ are

(a) $\bar{3}, \bar{4}, \bar{5}$ (b) $\bar{1}, \bar{2}, \bar{4}$
(c) $\bar{2}, \bar{4}, \bar{7}$ (d) $\bar{2}, \bar{4}, \bar{6}$

Question 18 In the group $G = \{2, 4, 6, 8\}$ under multiplication modulo 10, the identity element is

(a) 2 (b) 4
(c) 6 (d) 8

Question 19 The polynomial $f(x) = 2x^2 + 4$ is

(a) irreducible over $\mathbb{Q}$ (b) reducible over $\mathbb{Z}$
(c) reducible over $\mathbb{R}$ (d) irreducible over $\mathbb{C}$

Question 20 For which of the following values of n does there exists a field of order n

(a) 6 (b) 81
(c) 21 (d) 36

Question 21 Let $\frac{1}{z^2(z+1)}$. In the Laurent series expansion of f about $z = 0$, the principal part is

(a) $\frac{1}{z^2}$ (b) $\frac{1}{z^2} - \frac{1}{z+1}$
(c) $\frac{1}{z^3} - \frac{1}{z^2}$ (d) $\frac{1}{z^2} - \frac{1}{z}$

Question 22 The value of $\frac{1}{2\pi\iota}\int_{|z|=5} \frac{z}{(z+\iota)(z+2\iota)} dz$ equals

(a) 1 (b) 2
(c) π (d) 2π

Question 23 The radius of convergence of the power series $\sum_{n=0}^{\infty} n!x^n$ is

(a) ∞ (b) 1
(c) 0 (d) 1/2

Question 24 Let $X_i = \{1, 2, ..., n\}, i \in \mathbb{N}$ be a family of discrete topological spaces. The the product space $\prod_{i\in\mathbb{N}} X_i$ is

(a) connected but not compact (b) compact but not connected
(c) neither compact nor connected (d) both compact and connected

Question 25 Let $\mathbb{R}$ be the set of real numbers with the usual metric and let $\mathbb{Q}$ be the set of rationals. Then $[0,2] \cap \mathbb{Q}$ is

(a) compact and complete (b) not compact but complete
(c) neither compact nor complete (d) compact but not complete

Question 26 Let A be a 3×3 matrix and suppose that the matrix B is obtained from A by the following row operations: $R_1 \leftrightarrow R_2,\ R_3 \to R_3 + 5R_2$. If $det(A) = 8$, then $det(B)$ equals

(a) 8 (b) 16
(c) -8 (d) 0

Question 27 Let G be a group such that $o(G) = 11^2\ 13^2$. Then

(a) G has two 11-sylow subgroups, two 13-sylow subgroups and G is Abelian
(b) G has one 11-sylow subgroup, one 13-sylow subgroup and G is Abelian
(c) G has two 11-sylow subgroups, two 13-sylow subgroups and G is non-Abelian
(d) G has one 11-sylow subgroup, one 13-sylow subgroup and G is non-Abelian

Question 28 Let G be a non-Abelian group of order 125. The order of centre $Z(G)$ of G is

(a) 5 (b) 125
(c) 25 (d) 1

Question 29 The degree of the minimal splitting field of the polynomial $f(x) = x^p - 2$ over $\mathbb{Q}$, where p is an odd prime, is

(a) p (b) p^2
(c) $p^2 - p$ (d) $p - 1$

Question 30 The equation $u_{xx} + xu_{yy} = 0$ is

(a) elliptic for $x > 0, y \in \mathbb{R}$ and hyperbolic for $x < 0, y \in \mathbb{R}$
(b) elliptic for all $(x, y) \in \mathbb{R}^2$
(c) hyperbolic for all $(x, y) \in \mathbb{R}^2$
(d) hyperbolic for $x > 0, y \in \mathbb{R}$ and elliptic for $x < 0, y \in \mathbb{R}$

Question 31 The solution of $u_{xx} - 4u_{xy} + 4u_{yy} = 0$ is

(a) $u = f(y + 2x) + g(y + 2x)$
(b) $u = f(y - 2x) + g(y + 2x)$
(c) $u = f(y + 2x) + xg(y + 2x)$
(d) $u = f(y + 2x) - xg(y + 2x)$

Question 32 A sphere with radius a is moving with constant velocity U in a liquid which is otherwise at rest. The velocity potential $\phi(r, \theta)$ for the flow is

(a) $\frac{1}{2}Ua^3r^{-2}\cos\theta$
(b) $\frac{1}{2}Ua^3r\cos\theta$
(c) $\frac{1}{2}Ua^2r^{-2}\cos\theta$
(d) $\frac{1}{2}Ua^2r^2\cos\theta$

Question 33 When $n \to \infty$, the Binomial distribution can be approximated by

(a) Bernoulli distribution
(b) Uniform distribution
(c) Poisson distribution
(d) None of these

Question 34 What is the probability of getting two 'six' on rolling a die 4 times?

(a) $\frac{179}{1296}$
(b) $\frac{170}{1296}$
(c) $\frac{2}{1296}$
(d) $\frac{171}{1296}$

Question 35 If the primal linear program has no solution, then dual of the problem

(a) Has either no solution or is unbounded
(b) Has unbounded solution
(c) Has an optimal solution
(d) None of these

Question 36 The function $f(x) = \frac{1}{x-a}\sin\frac{1}{x-a}$ has

(a) Discontinuity of first kind at $x = 0$
(b) Discontinuity of second kind at $x = 0$
(c) Continuity at $x = 0$
(d) None of these

Question 37 The expected value of the random variable X whose probability density is given by

$$f(x) = \begin{cases} \frac{x+1}{8} & 2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) $\frac{37}{6}$ (b) $\frac{37}{12}$

(c) $\frac{37}{18}$ (d) $\frac{37}{24}$

Question 38 The curve on which the functional $I(x, y, y') = \int_0^1 ((y')^2 + 12xy)dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremized is

(a) $y = x^2$ (b) $y = x^3$

(c) $y = x^4$ (d) $y = x + x^2$

Question 39 The partial differential equation $xy\frac{\partial z}{\partial x} = 5\frac{\partial^2 z}{\partial y^2}$ is classified as

(a) elliptic (b) parabolic

(c) hyperbolic (d) circle

Question 40 A block of mass M and contact area A slides an inclined plane with friction, covering a distance L in time T. How much time does it take another block with the same mass and composition, but contact area $2A$, to slide down the same length

(a) $\sqrt{T}$ (b) T

(c) T^2 (d) $2T$

Question 41 Let A be the set of irrational numbers between 0 and 1. Then the Lebesgue measure of A is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) ∞

Question 42 Let X be the set with at least two elements. Let τ and τ' be two topologies on X, such that $\tau' \neq \{\emptyset, X\}$. Which of the following condition is necessary for the identity function $id : (X, \tau) \to (X, \tau')$ to be continuous

(a) $\tau \subseteq \tau'$ (b) $\tau' \subseteq \tau$

(c) no condition on τ and τ' (d) $\tau \cap \tau' = \{\emptyset, X\}$

Question 43 The set $[0,1] \times (0,1) \subset \mathbb{R}^2$ is

(a) open	(b) closed
(c) compact	(d) connected

Question 44 If A is a 5×5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A, each with algebraic multiplicity 2, then the determinant of A is equal to

(a) 2	(b) 3
(c) 120	(d) 180

Question 45 Which of the following is not an orientable surface

(a) torus	(b) cylinder
(c) mobius strip	(d) sphere

Question 46 Total number of positive integer solutions to the equation $(x_1+x_2+x_3)(y_1+y_2+y_3+y_4) = 15$ is

(a) 1	(b) 2
(c) 3	(d) 4

Question 47 A polynomial of odd degree with real coefficients must have

(a) At least one real root	(b) No real root
(c) Only real roots	(d) At least one root which is not real

Question 48 The number of subfields of a field of cardinality 2^{100} is

(a) 2	(b) 4
(c) 9	(d) 100

Question 49 Let $f : X \to X$ be a function such that $f(f(x)) = x$ for all $x \in X$. Then

(a) f is injective and surjective	(b) f is injective but not surjective
(c) f is surjective but not injective	(d) f is neither surjective nor injective

Question 50 The minimum value of $x^5 + y^5$ for $x + y = 2$ with $x, y \geq 0$ is

(a) No minimum value	(b) 2
(c) 32	(d) 0

x – x – x